Maximizing Sustainability Improvement

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Draft prepared for the Conference **Investing Resource Rents: A look at resource economics after 40 years of Hartwick's Rule**, Queen's University, Kingston, Canada. October 20-21, 2017.

– Work in progress–

Abstract

The highest level of utility that could be sustained forever given the endowment of the economy is given by the maximin value. If this level is judged too low, sustainable development may be interpreted as a requirement to improve the sustainable level, for a further stage of development. Such an improvement comes, however, at a cost in terms of current utility, which must be reduced below the maximin value. We investigate investment strategies to improve sustainability when current generation makes such a sacrifice. We then discuss the induced trade-offs among current sacrifice, sustainability improvement, and the long-run sustained utility. The general results are illustrated in two canonical, stylized economies: the simple fisery and the Dasgupta-Heal-Solow model.

Key words: sustainable development, sacrifice, growth, maximin value

JEL Code: O44; Q56.

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1 Introduction

A *sustainable development* describes growth out of poverty toward a developed state that can be sustained for the *very long run* (Solow, 1993). In an efficient economy, growth or development entails the diversion of resources from consumption[1] by the current generation to investment that will increase productivity in the future. Although the implications for a poor society are not usually stressed, a policy proposal of greater current investment and less consumption has been advanced in several economic models that assume efficiency in the attainment of sustainability. Optimal growth theory, however, neither specifies the extent of sacrifice envisaged, nor values growth *per se* in the definition of welfare, although growth is considered good. The current standard of living in a less developed country may be so low that one cannot contemplate reducing it. Sacrificing the interests of the present may be inconsistent with the Brundtland Report's (World Commission on Environment and Development, 1987) famous dictum on sustainability, which balances and protects the interest of the present as well as the future. One way to avoid this invidious trade-off is to assume that, while the present generation is poor, there is some possibility of improvement from the base of the present. Llavador, Roemer and Silvestre (2011), for example, find that sustainable consumption for the USA was higher than actual consumption in 2000. A possible reason is inefficiency. As Llavador *et al.* indicate, the long-term solution is to address the inefficiency, not necessarily to invest more in the present. In the present paper, we are not focusing on inefficiency, but on investment.

Rawls (1971) pointed out that optimal growth (under some utilitarian objective) may unreasonably require too much savings from poor generations for the benefits of their wealthier descendants. The maximin approach is an alternative to optimal growth in the definition of sustainability (Cairns and Long, 2006 ; Fleurbaey, 2015). The objective is to follow the economic path for which the utility of the

[1]For the sake of definiteness, utility or the standard of living of the society is frequently referred to herein as its level of consumption, broadly defined.

poorest generation is the highest (Solow, 1974). In a *regular* maximin problem, the maximin path results in constant utility over the indefinite future (Burmeister and Hammond, 1977). The main criticism against applying maximin as a normative objective for sustainability is that, if a planner decides to apply the criterion immediately in a poor economy, future generations may be mired in a "poverty trap," involving continuing levels of the standard of living equal to the low level of the present: poverty may be sustained. The criticism implies that the present generation is considered to be at a level of poverty that is so dire that the future must be rescued from it. Rawls himself acknowledged that economic growth may be necessary to reach a state of the economy in which material resources are sufficient to implement a "just society." He pointed out that the maximin principle must be modified to allow for economic growth.We examine a modification to the maximin program as a response to this criticism. Since, for a regular maximin program, the constant utility path is a Pareto-efficient solution, we show that for sustainable growth to occur the standard of living of the present must be reduced to an *even lower* level than that of the poverty trap. There is tentative evidence that consumers prefer wage or consumption profiles that increase through time (Lowenstein and Sicherman, 1991; Frank and Hutchens, 1993), suggesting that the current sustainable consumption may seem too low, or that growth is desirable *per se*. If such preferences can be applied to a whole society, the current generation may be willing to reduce current consumption with respect to the sustainable one, to make sustained growth possible. It has a flavor similar to the preference for "sustained improvement" proposed by Pezzey (1997). It is of interest to study the trade-off between this sacrifice and the prospect for growth it offers.

The present paper addresses the implications of a conscious choice by a society between current sacrifice and sustainability improvement. For growth to be sustainable, the development path followed by the economy must be within environmental and technological constraints. Sustainable development means that an acceptable standard of living is reached in the long run and then sustained. The issue is how to grow out of "poverty", to improve what can be sustained. We examine the conditions for growth to be sustainable and the conditions for the sacrifice of present generations to improve sustainability. Whatever the objective of the society and the growth pattern it follows, the maximin value of utility is an indicator of sustainability, characterizing the dynamic limit to growth. The maximin value has a clear meaning in terms of the sustainability of the society along the chosen trajectory. If consumption is lower than the maximin value over some interval and the output so freed up is invested in long-term productive capacity, the sustainable level of consumption can be permanently increased. Conversely, consumption over a current interval can be increased at the expense of investment and hence of sustainable consumption in the future. Even though a maximin policy may not be being pursued, at any economic state a maximin level of utility can be determined by solving the maximin problem for the stocks at that state. The evolution of this maximin value along any trajectory plays a fundamental role in the sense that it is an indicator of what is sustainable. Current decisions reduce what is sustainable if the maximin value decreases and result in sustainability improvement if the maximin value increases (Cairns and Martinet, 2014). More specifically, if the level of utility is lower than the maximin level on an interval, both the attainable maximin level of utility of the economy and current utility can increase through time. Utility can continue to grow as long as it stays below the dynamic maximin indicator. Once utility catches up with the indicator's level, utility can be sustained only at the maximin level. In this way, utility *growth* can be maintained until the eventual, *sustained level* of utility is reached. There are trade-offs among present utility, growth and long-run sustained utility, given technological and natural conditions.

We show that, even if maximin utility is not the pursued social objective, an investment strategy maximizing what we call *sustainability improvement*, results in sustainable growth if and only if utility is lower than the maximin value.

Our study is motivated by two model economies that have been prominent in the study of sustainability. In a simple fishery, a fish stock is harvested and consumed directly. Open access leads to over-exploitation. At any point, however, the society is assumed to be in a position to choose a level of employment in the industry, and hence of forbearance in exploiting the stock. At the beginning of the program the stock is at a low level (is "overfished") and the society wishes to rebuild its stock by limiting current consumption. In the steady state, the harvest is equal to natural growth and is thus sustained.

In the *Dasgupta-Heal-Solow* (DHS) model of an economy dependent on manufactured capital and an essential, non-renewable resource (Dasgupta and Heal, 1974; Solow, 1974), sustaining consumption at a constant level requires that investment in manufactured capital offset the depletion of the resource (Hartwick, 1977). A deviation downward from that possible constant-consumption path can allow for growth at a parametric rate through investment (d'Autume and Schubert, 2008). The economy can choose from many different paths of sustained development. Overshooting the sustainable path is also possible in the model.

Each of the two canonical models addresses a fundamental issue in environmental economics. Each implies that growth is subject to environmental constraints. Open access in the fishery leads to a tragedy of the commons. The DHS economy illustrates the fact that sustaining an economy may not involve a steady state. Each of open access and growth can lead to unsustainability and to a poverty trap.

2 Sustainability improvement

2.1 Economic model and maximin value

Consider vectors of capital stocks $X \in \mathbb{R}_+^n$ (either manufactured, natural or human capital) and decisions c (consumption, extraction, etc.) within the set $C(X) \subseteq \mathbb{R}^p$ of admissible controls at state X. The transition equation for each state variable X_i, $i = 1, \ldots, n$ is given by

$$\dot{X}_i = F_i(X, c) \ ,$$

where the functions F_i can represent technologies or natural resources dynamics. The instantaneous utility $U(X, c)$ may depend on the state and the decisions.

A maximin path maximizes the standard of living of the poorest generation, looking forward from the present (Cairns and Long 2006). What is *sustained* (supported from below) along a feasible path of the economy is the minimum level of utility of any generation over the very long run. The maximum attainable such minimum level, or the maximin level, is what is *sustainable*. Formally, the maximin value of any state X is denoted by $m(X)$ and given by

$$\begin{aligned} m(X) \quad &= \quad \max \bar{U} \\ \text{s.t.} \quad & U(X(t), c(t)) \geq \bar{U} \ , \forall t \geq 0 \\ & \dot{X}_i = F_i(X, c) \\ & X(0) = X \ . \end{aligned} \tag{1}$$

If the economy pursues the maximin objective in a *regular* maximin problem, the standard of living remains constant over the indefinite future (Burmeister and Hammond, 1977; Cairns and Long, 2006), which may not be a desired normative objective.

The maximin value is interpreted by Cairns and Martinet (2014) as a dynamic limit to sustainable growth, i.e., as some generalized carrying capacity for the economy. Its evolution over time informs, along any path, on the way current decisions modify the level of utility that could be sustain from current state. This evolution is given by net investment at maximin accounting prices, which is defined for a given current economic state X and any admissible vector of decisions c as follows

$$M(X,c) = \dot{m}(X)\,|_c = \sum_{i=1}^{n} \dot{X}_i \frac{\partial m(X)}{\partial X_i} = \sum_{i=1}^{n} F_i(X,c) \frac{\partial m(X)}{\partial X_i}.$$

The maximin shadow values at state X, $\frac{\partial m(X)}{\partial X_i}$, depend only on the current state and not on the economic decisions. They are thus the same whatever the trajectory determined by the functions $F_i(X,c)$.

Our analysis is based on a modification to the maximin program that allows for growth. More specifically, we are interested in the effect of a current sacrifice (i.e., a consumption lowering utility with respect to the maximin value) on sustainability improvement.

2.2 Instantaneous maximization of sustainability improvement

Let us introduce formally the way a local deviation from the maximin problem affects the maximin value. In a maximin problem, let the co-state variables of the capital stocks be denoted by μ_i, $i = 1, \ldots, n$. The objective of a maximin problem is mathematically expressed as the maximization of the Hamiltonian $H(X,c) = \sum_{i=1}^{n} \mu_i \dot{X}_i$, subject to the constraint $U(X,c) \geq m(X)$ (Cairns and Long, 2006). It is equivalent to maximize the Lagrangean:

$$\begin{aligned} L(c,X,\nu) &= H(X,c) + \nu\,(U(X,c) - m(X)) \\ &= \sum_{i=1}^{n} \mu_i \dot{X}_i + \nu\,(U(X,c) - m(X)) \ , \end{aligned}$$

where ν is the dual variable of the equity constraint. Note that the term $\nu\,(U(X,c) - m(X))$ satisfies the complementarity slackness condition, and is always equal to zero. Cairns and Long (2006, Proposition 1) show that the co-state variables of a maximin problem are equal to the derivatives of the maximin value function with respect to the state variables: $\mu_i = \frac{\partial m}{\partial X_i}$. Therefore,

$\dot{m}(X) = \sum_{i=1}^{n} \frac{\partial m(X)}{\partial X_i} \dot{X}_i = \sum_{i=1}^{n} \mu_i \dot{X}_i$, and the previous Lagrangean can be written as

$$L(c, X, \nu) = \dot{m}(X) + \nu \left(U(X, c) - m(X)\right) \ .$$

The maximin problem is thus tantamount to maximizing the net investment at maximin shadow values, i.e., $\dot{m}(X)$, subject to the constraint that consumption is no less than the maximin value.

In a maximin problem, the level of the equity constraint is increased as much as possible. Hartwick's (1977) rule is that, at the maximum, $H(X, c) = \dot{m}(X) = \sum_{i=1}^{n} \mu_i \dot{X}_i = 0$. The minimal utility is increased up to the point at which the maximal net investment is nil. We deviate from this maximin problem in the sense that we do not maximize the minimal utility over time. However, we examine how the net investment at maximin shadow values evolves.

Consider a "deviation" from the maximin problem, in the sense that the sustainability constraint is relaxed and current consumption is allowed to be lower than the maximin level. We explore the case in which the social planer maximizes net investment accounted at the maximin shadow values subject to a given, targeted utility level $\bar{U}(t)$. This is equivalent to maximizing the modified Lagrangean,

$$\tilde{L}(c, X, \tilde{\nu}) = \dot{m}(X) + \tilde{\nu} \left(U(X, c) - \bar{U}(t)\right) \ .$$

That is to say, the program is to maximize $\dot{m}(X)$ subject to a modified constraint. The modified complementarity slackness condition is again equal to zero.

Definition 1 Instantaneous maximization of sustainability improvement. *The resource-allocation mechanism is said to maximize sustainability improvement at current time if decisions c maximize the increase of the maximin value subject to the given current utility:*

$$\begin{array}{lll} c & \text{maximizes} & M(X, c) = \sum_{i=1}^{n} F_i(X, c) \frac{\partial m(X)}{\partial X_i} \\ & \text{s.t.} & U(X, c) \geq \bar{U}(t) \end{array} \qquad (2)$$

The interpretation of this resource allocation mechanism is that current generation makes a sacrifice and increases the limit to growth as much as possible, given its current utility. This investment strategy only depends on the current generation's decisions. We shall see that it may, however, induce intertemporal inefficiencies. In particular, it does not maximize the long-run utility given the initial consumption and some growth pattern, or maximize the initial consumption given the growth pattern and long-run consumption. The definition of such an efficient sustainable growth path is a task for future research. We shall provide some insights into this question at the end of the section.

2.3 Trade-off between current utility and the sustained utility reached in the long run

The previous analysis only considers an instantaneous deviation from a maximin path, whereas sustainaible development is more likely to be an intertemporal problem, with an increase of the maximin value and a sustained long-run utility.

In the tradition of Ramsey's (1928) model of undiscounted utility, some authors assume that growth leads the economy toward a bliss utility level. (See d'Autume and Schubert (2008) for an analysis of the DHS model in this framework.) The exogenous bliss level of utility coincides with the (green) golden rule and is approached asymptotically. A sustainable development pattern may, however, lead to a lower level of utility, just as the modified golden rule of discounted utility corresponds to a lower development than the golden rule.

We can define sustained development as follows. Utility can increase according to some growth pattern[2] as long as it is lower than the maximin value, which rep-

[2]Growth theorists have specified as parameters certain variables that could have been modeled as choices. Among them are a constant savings ratio, a constant capital-output ratio, balanced growth, or a constant "bliss" level of utility to be reached in the long-run. Holding a variable constant in this way has simplified complicated dynamic problems and has allowed for many revealing analyses. Following these tentatives in growth theory and in positive economics, we shall assume in our examples that the economies are not pursuing a specific objective but rather

resents generalized economic carrying capacity. When utility catches up with the maximin value, the economy has to stop growing and to follow the maximin path starting from the economic state reached. What matters is thus how the maximin value evolves. As the maximin value is dynamic, there is a trade-off between the initial level of utility, the pursued growth pattern, and the duration of the growth period or equivalently the level of sustained utility that is reached in the long run. If one assumes a given growth pattern combined with the instantaneous optimization problem of maximizing sustainability improvement, it is possible to determine endogenously the long-run, sustained level of utility. This makes it possible to exhibit the trade-off between social utility at time t and the ultimate utility reached in the long run.

The next section examines this issue in the two canonical models. We show that in our fishery model the bliss level is the MSY, which is not necessarily the long-run level achieved by the society. In the DHS model there is no exogenous bliss level and the long-run consumption is also endogenous. As a consequence, such long-run sustained utility level could be a social choice.

Even if the analysis in this paper makes no assumption on the growth pattern (in the sens that it is considered as given) and focuses on current decisions, a potential research question could be to determine the resource allocation mechanism that maximizes the long-run utility given a current level of utility and a growth pattern,

a parametric policy that seems plausible, for example, constant employment or constant growth. Growth of consumption at a constant rate can be considered to be a generalization of a sustained path in that its "distribution over time has some definite, standard shape" (Hicks, 1946: 184).

i.e., a given path of utility $\bar{U}(t)$. The problem would take the following form:[3]

$$\begin{aligned} \max_{c(\cdot),T} \quad & \int_0^T \dot{m}(K,S,r,c)dt \\ s.t. \quad & U(X(t),c(t) = \bar{U}(t) \\ & \dot{X}(t) = F(X(t),c(t)) \\ & X(0) = X_0 \end{aligned} \tag{3}$$

This is equivalent to maximizing the reached sustainable level $m(X(T))$. Such a research question would introduce the idea of intertemporal efficiency. It, however, requires strong assumptions on the growth pattern, or underlying objective of the successive generations. The instantaneous maximization of sustainability improvement makes no assumptions regarding future generations, and considers only the potential room for growth bequeathed to them by the current generation. Optimal investment strategies may differ from the sufficient condition of maximized sustainability improvement. It is sure, however, that the general idea of our results will hold: sustainable growth requires current utility to be lower than the maximin level *and* investment has to be such that the maximin value is increasing.

[3]An additional constraint could be added to impose a smooth utility path, with $m(X(T)) = U(X(T),c(T)$.

3 Case studies

We apply the proposed approaches to two case studies. First, we use a simple model with a single decision to illustrate sustained development with a given growth pattern and the associated welfarist considerations on the trade-off between current sacrifice and long-run, sustained utility. For this purpose, we consider the simple fishery model. We then introduce the more sophisticate Dasgupta-Heal-Solow model of production and consumption with a manufactured capital stock and a nonrenewable resource stock. Although this canonical model is still simple, it encompasses sufficient elements to allow us to discuss the cases of maximization of sustainability improvement.

3.1 The Simple Fishery

3.1.1 The model

Denoting the natural rate of growth of the fish stock $S(t)$ at time t by $S(t)[1-S(t)]$, fishing effort by $E(t)$ and the consumption (harvest) of the resource by $C(t) = S(t)E(t)$, we study the following simple model of the evolution of the stock:[4]

$$\dot{S}\,(t) = S(t)\,[1 - S(t)] - S(t)E(t). \tag{4}$$

We assume that the effort E belongs to the interval $[0, 1]$. The open-access regime has $E(t) = E_0 = 1$.

In this model, the highest sustainable level of consumption is called the "maxi-

[4]This model is often written using the parameters r, S_{sup} and q to represent the natural growth rate of the resource, its carrying capacity, and the catchability of the resource, so that

$$\dot{S}(t) = rS(t)\left(1 - \frac{S(t)}{S_{sup}}\right) - qS(t)E(t).$$

In our model, without loss of generality we define units of time, of effort and the resource such that $r = 1$, $S_{sup} = 1$, and $q = 1$. The expressions are less cumbersome, but one must be careful to keep track of the units in which the variables are measured.

mum sustainable yield" (MSY). Its value is

$$C_{MSY} = \max_{S} \left[S\left(1 - S\right)\right] = \tfrac{1}{4}.$$

The associated stock is $S_{MSY} = \frac{1}{2}$ and the equilibrium level of effort is $E_{MSY} = \frac{1}{2}$. The MSY stock is a benchmark for the study of both ecological and economic overexploitation.[5] If the initial state S_0 is lower than that associated with the MSY, the maximin criterion (1) leads to a constant harvest in equilibrium, $C(t) = S_0\left(1 - S_0\right)$. If the initial state is above the MSY level, the maximin value is the MSY harvest. We thus define the maximin value, given the state S at time t, of this economic system as

$$m(S) = \begin{cases} S_{MSY}\left(1 - S_{MSY}\right) & \text{if } S > S_{MSY}, \\ S(1 - S) & \text{if } S \leq S_{MSY}. \end{cases} \tag{5}$$

If $S \leq S_{MSY}$, the level of effort, E^{mm}, on a maximin path is such that the harvest is equal to the natural growth, so that $E^{mm}S = S\left(1 - S\right)$, or $E^{mm} = 1 - S$.

Let the initial state S_0 be lower than the MSY biomass, i.e., $S_0 < S_{MSY}$, as may have occurred if the economy has been facing a "tragedy of the commons" for some time because of an initial open access to the resource. The stock can be considered to be over-exploited, or vulnerable to over-exploitation and a poverty trap, with a low sustainable (maximin) level of exploitation. If the stock recovers from over-exploitation, the maximin value can increase.

3.1.2 Trade-off between current sacrifice and instantaneous sustainability improvement

Let us first consider the case in which current generation decides to reduce consumption at the current time to improve sustainability.

In this simple fishery model, for a capital stock $S \leq S_{MSY}$, maximin improvement is given by

$$M(S, E) = \frac{dm(S)}{dt} = \frac{dm(S)}{dS}\dot{S} = (1 - 2S)(S(1 - S) - SE)\;. \tag{6}$$

[5]For sake of simplicity, we do not consider the cost of effort. As a result, the maximum economic yield (golden rule) coincides with the maximum sustainable yield.

Eq. (6) gives the sustainability improvement as a function of the actual fishing effort E and current stock S. This equation can be interpreted as the product of two effects: a current stock effect $(1-2S)$ and a sacrifice effect $(S(1-S)-SE)$. The stock effect corresponds to the marginal productivity of the stock and measures the additional productivity of a marginal investment (a marginal reduction of consumption). The sacrifice effect corresponds to the foregone consumption with respect to the maximin level. Along a maximin path, one would have no sacrifice, with the full consumption of the produced resource and a stationary resource stock, i.e., $SE = S(1-S)$ and $\dot{S} = 0$. The greater the sacrifice with respect to current maximin value, the larger the sustainability improvement. Also, the smaller the current resource stock, the larger the maximin improvement for a given sacrifice.[6]

Eq. (6) is linear in the fishing effort, and thus in the current utility (or symmetrically, current sacrifice). It is positive and declining for $E \leq E^{mm}$. If the willingness to sacrifice for sustainability improvement is deacreasing when the maximin improvement increases, we can assume that there is a well-defined optimal level of sacrifice for current generation.

Studying the recursive pattern of such optimal sacrifices is a question of interest that we plan to investigate.

3.1.3 Long-run recovery under constant effort

Let a level of effort be chosen and remain constant at the level $E_0 \in]0,1[$. Such a strategy could aim at increasing the available resource and sustainable consumption while maintaining an acceptable level of employment in the fishery. Consumption is given by $C(t) = E_0 S(t)$ and the dynamics of the exploited resource becomes

$$\dot{S}(t) = S(t)\left(1 - E_0 - S(t)\right). \tag{7}$$

[6] At or above the MSY stock $S_{MSY} = 1/2$, sacrifices have no effect.

Along this trajectory, the stock evolves as

$$S(t) = \frac{1}{\frac{1}{1-E_0} + \left(\frac{1}{S_0} - \frac{1}{1-E_0}\right) e^{-(1-E_0)t}}. \tag{8}$$

The system tends toward a limit, $S_\infty = 1 - E_0$.

The rule of constant effort completely determines the trajectory of this fishery. By equation (5), when $S \leq S_{MSY}$, the maximin level of effort is given by $E^{mm}(S) = 1 - S$. This level of effort maintains the stock at a stationary level that may correspond to a "poverty trap." In order to recover from a period of overfishing, society must harvest less than the maximin harvest $m(S) = S(1-S)$ so that the stock can grow and the maximin value function can increase along the trajectory. This feature of the problem illustrates that there is no "free lunch" for the future. Current effort must be less than $E^{mm}(S_0)$, and consumption less than $C^{mm} = S_0(1 - S_0)$.

Under a strategy of constant fishing effort, with $E(t) = E_0 < 1 - S_0 = E^{mm}(S_0)$, fish consumption increases with the stock size. Fig. 1 depicts the following trajectories through time, beginning at the stock $S_0 = 0.1$:

- The natural growth of the stock (without harvesting).[7]
- The growth of the resource stock with constant fishing effort $E_0 = E^{MSY} = \frac{1}{2}$. The stock tends toward S_{MSY}. This trajectory is labeled "stock recovery."
- The trajectory of the maximin value along the trajectory for $E_0 = \frac{1}{2}$. The maximin value increases toward the MSY level.
- The consumption pattern, which increases as the stock increases and catches up to the maximin value. Consumption tends toward the MSY.

[7] With no consumption ($C(t) = 0$, i.e., $E(t) = 0$), the dynamics of the resource stock is given by

$$S(t) = \frac{1}{1 + e^{-t}(\frac{1}{S_0} - 1)}. \tag{9}$$

The stock recovers faster, but the present generation does not consume at all.

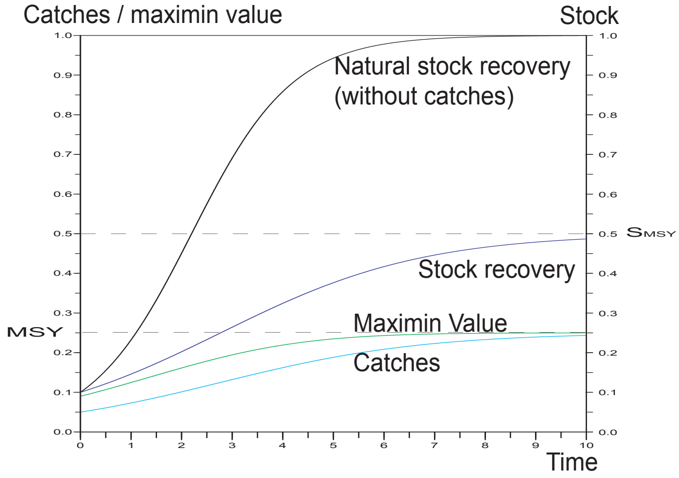

Figure 1: Evolution of the maximin value function along a constant effort trajectory leading to Maximum Sustainable Yield

We stress that the recovery of the fishery (and thus the increase in consumption) is possible only because consumption is lower than the maximin level at all times. The long-run consumption depends on the reduction of the present consumption, the constant fishing effort being between the maximin value $E^{mm}(S_0) = 1 - S_0$ and the MSY value $E^{MSY} = \frac{1}{2}$. A lower fishing effort, and hence current consumption, entails a higher long-run consumption.[8] Fig. 2 presents the trajectories of maximin value and catches for three different recovery strategies (for three different effort levels) with, again, an initial fish stock $S_0 = 0.1$. For this stock, the initial maximin value is $m(S_0) = S_0(1 - S_0) = 0.1(1 - 0.1) = 0.09$.

- The first strategy (trajectories denoted by $MV_{0.9}$ and $C_{0.9}$) corresponds to a constant fishing effort $E_0 = E^{mm} = 0.9$. At this effort level, the stock is in equilibrium at the initial value, i.e., $S_\infty = S_0 = 0.1$. The harvest is equal to the maximin value from the initial stock at all times.

[8]Effort levels below $\frac{1}{2}$ are not considered as they would result in lower catches both for present and future generations.

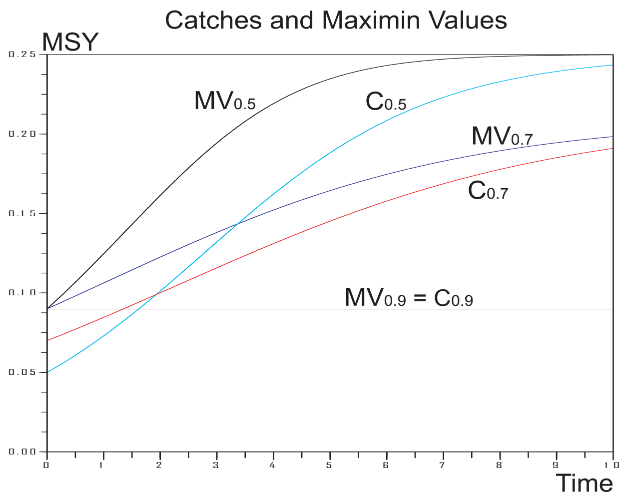

Figure 2: Sensitivity analysis (with respect to the constant effort level)

- The second strategy (trajectories denoted by $MV_{0.7}$ and $C_{0.7}$) corresponds to a constant fishing effort $E_0 = 0.7 < 0.9$. The fish stock increases asymptotically toward a limit, $S_\infty = 1 - E_0 = 0.3$ (not represented on the figure). The harvest increases toward the maximin harvest for this limit, $S_\infty(1 - S_\infty) = 0.21$, which is lower than the MSY.

- The third strategy (trajectories denoted by $MV_{0.5}$ and $C_{0.5}$) is that depicted in Fig. 1, with the fishing effort set constant at the MSY equilibrium effort, 0.5. The maximin value increases asymptotically toward the MSY value and the harvest increases toward the MSY, which is 0.25.

There is a non-linear relationship between C_0 and C_∞ which is determined by the chosen (constant) effort level. Recovery effort belongs to $[E^{MSY}, E^{mm}(S_0)]$. If the effort is small and equal to E^{MSY}, present consumption is low ($C_0 = E^{MSY} S_0$) and the limiting consumption is the MSY. If the effort is equal to $E^{mm}(S_0)$, the stock remains at the initial level S_0, and the present and limiting consumption are equal. (There is no growth.) This is the maximin path, sometimes criticized as possibly entrenching a poverty trap. Intermediate cases are defined according to the relationship

$$C_\infty = \lim_{t\to\infty} E_0 S(t) = E_0 (1 - E_0) = \frac{C_0}{S_0}\left(1 - \frac{C_0}{S_0}\right), \tag{10}$$

for $C_0 \in [S_0/2, S_0]$, i.e., for $E_0 \in [1/2, 1]$. The possibility frontier between present and future consumption is described by Fig. 3.

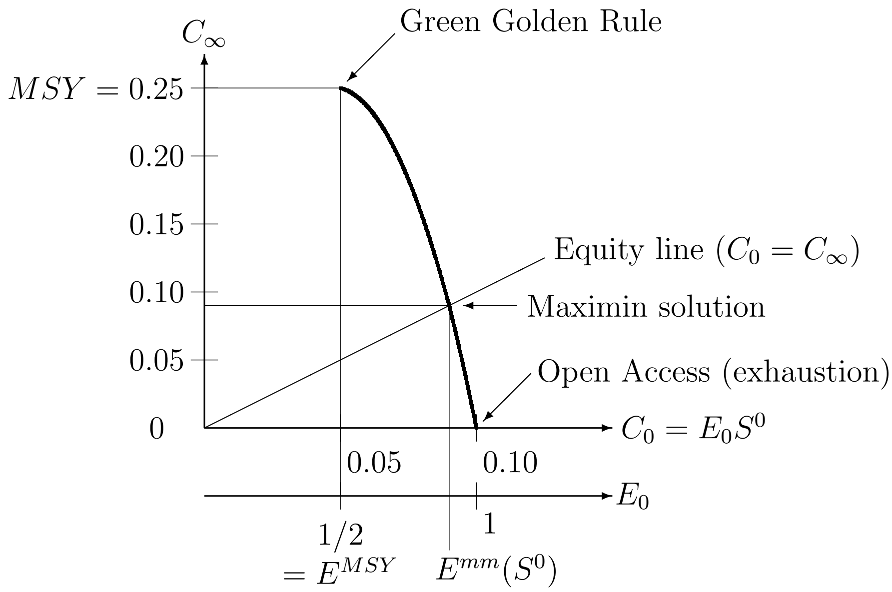

Figure 3: Trade-off between present consumption and long-run consumption in a fishery with constant effort and $S_0 = 0.1$.

Any pair (C_0, C_∞) that is achievable with constant effort belongs to this frontier. Social preferences between present and future consumption can be given by a function $\Psi(C_0, C_\infty)$, which can be maximized along the frontier. Several particular solutions are represented in Fig. 3, including the Green Golden Rule (Chichilnisky, Heal and Beltratti, 1995) corresponding to $\Psi(C_0, C_\infty) \equiv C_\infty$; myopic behavior from open access, corresponding to $\Psi(C_0, C_\infty) \equiv C_0$; and the maximin, corresponding to $\Psi(C_0, C_\infty) \equiv [\min(C_0, C_\infty)]$. Once initial and final consumption are chosen, the (logistic) growth rate is endogenous under the assumption that effort is constant.

3.2 The Dasgupta-Heal-Solow Model

3.2.1 The model

Consider a society that has stocks of a non-renewable resource, S_0, and of a manufactured capital good, K_0, at its disposal at time $t = 0$. It produces output (consumption c and investment $\dot{K}$) by use of the capital stock and by depleting the resource stock at rate

$$r(t) = -\dot{S}(t), \tag{11}$$

according to a Cobb-Douglas production function:

$$c + \dot{K} = F(K, r) = K^{\alpha} r^{\beta}, \text{ with } 0 < \beta < \alpha, \text{ and } \alpha + \beta \leq 1 . \tag{12}$$

This model has been used by many authors to study the implications of exhaustibility of an essential resource, including how to sustain consumption in the face of exhaustibility. If the discounted-utility criterion is applied to this economy, consumption decreases asymptotically toward zero (Dasgupta and Heal 1974, 1979). Analysis of how consumption can be sustained requires a different approach from discounted utilitarianism. For given levels of the capital and resource stocks, Solow (1974) and Dasgupta and Heal (1979) show that the maximal consumption that the economy can sustain, the *maximin* level, is given by

$$m(S, K) = (1 - \beta)(\alpha - \beta)^{\frac{\beta}{1-\beta}} S^{\frac{\beta}{1-\beta}} K^{\frac{\alpha-\beta}{1-\beta}} . \tag{13}$$

Since this aggregate of the two stocks measures the capacity of the economy to sustain the standard of living $m(S, K)$ for the long term, it is an index of sustainability. It is an increasing function of both stocks.

3.2.2 Trade-off between current sacrifice and instantaneous sustainability improvement

Here again, we start considering the case in which current generation decides to reduce consumption at the current time to improve sustainability.

In the DHS model, maximin improvement is given by

$$M(K,S,c,r)=\frac{dm(K,S)}{dt} = \frac{\partial m(K,S)}{\partial K}\dot{K}+\frac{\partial m(K,S)}{\partial S}\dot{S}$$
$$= \frac{\partial m(K,S)}{\partial K}(K^{\alpha}r^{\beta}-c)-\frac{\partial m(K,S)}{\partial S}r \qquad (14)$$

Contrary to the single-stock problem of the fishery, a sacrifice does not necessarily entail sustainability improvement in the DHS model. For a level of consumption $c < m(S,K)$, this expression may be either positive or negative depending on the extraction level, and thus production and investment.

The level of natural resource extraction maximizing $\dot{m}$, conditional on the consumption level, is given by the following extraction rule $\hat{r}(K,S)$:[9]

$$\hat{r}(K,S)=(\alpha-\beta)^{\frac{1}{1-\beta}}S^{\frac{1}{1-\beta}}K^{-\frac{1-\alpha}{1-\beta}}\ . \qquad (15)$$

This feedback rule is the same as the one along the maximin path. At any given state, maximizing sustainability improvement entails producing the same as for the maximin path at the current state, and investing any amount of capital freed up by a sacrifice of current consumption.

Under this strategy of maximizing sustainability improvement through the extraction rule $\hat{r}(K,S)$, maximin improvement can be expressed as a function of the state variables and the consumption only:

$$M(K,S,c,\hat{r}(K,S))=\frac{\partial m(K,S)}{\partial K}(K^{\alpha}\hat{r}^{\beta}-c)-\frac{\partial m(K,S)}{\partial S}\hat{r}\ .$$

This expression is linear in the current consumption (or symmetrically, current sacrifice), just as in the fishery model. It equals zero when the consumption equals the maximin level (and net investment is nil), and is positive whenever the consumption is lower than the maximin level, corresponding to a positive net investment. The marginal effect of the sacrifice is proportional to the shadow value of the capital stock.

[9]Mathematical details are in the appendix.

3.2.3 Trade-off between current consumption, growth rate and long-run consumption

Sustained development in the DHS model can be represented as follows. The economy can grow from an initial level of consumption c_0 that is less than the sustainability indicator provided by the maximin value $m(S_0, K_0)$, according to some resource-allocation mechanism. This choice allows for sustainability improvement and growth.

Illustrative resource-allocation mechanism. For the sake of simplicity, let consider that the society initially pursues consumption growth at a constant rate $g > 0$.[10] We assume that the consumption side of the economy is determined by the constant growth rate pattern, with consumption

$$c(t) = c_0 e^{gt} \tag{16}$$

and complete the resource–allocation mechanism by assuming that the current generation maximizes sustainability improvement, though the feedback extraction rule $\hat{r}(K, S)$.

The limit to growth. There is a limit to the time for which growth can be supported at rate g. If growth is pursued without considering the maximin value and its evolution, consumption overshoots the sustainable level at some time, and sustainability collapses, as illustrated in Fig. 4.

To avoid this unsustainable type of trajectory, the economy must switch at some time T from the exponential growth path to a maximin path characterized by constant consumption $c_\infty \equiv m(S(T), K(T))$. In fact, the long-run level of consumption is endogenous, and is defined as the time at which consumption catches up with the maximin level $m(S(T), K(T))$.[11] Fig. 5 illustrates two sustained-development

[10] Any other growth pattern could have been used for illustrative purpose. A constant rate of growth seems a natural choice as it has been used by other authors, such as Lowenstein and Sicherman (1991) and Frank and Hutchens (1993), mentioned above. The World Bank (2011) assumes that consumption changes at a constant rate in its study of sustainable development and Llavador *et al.* (2011) provide that utility changes at a constant rate.

[11] Another possibility is to imagine a path for which the rate of growth smoothly approaches the

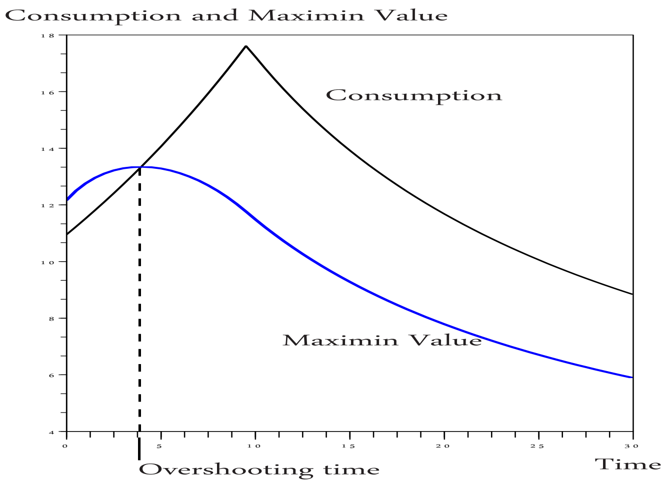

Figure 4: Exponential consumption and Maximin value function

paths, with different initial consumption and growth rates. The long-run, sustained consumption is different.

Sustained growth at rate $g > 0$ demands that $c_0 < m(S_0, K_0)$. The growth rate and the duration of the growth period are linked to the initial consumption and the long-run, sustained consumption. If two of the four are given, the two others can be derived. For any initial pair $(S_0, K_0) > 0$, there is a maximin level of consumption $m_0 = m(S_0, K_0) > 0$ given by equation (13). Also, for any initial pair of stocks it is possible at time $t = 0$ to choose any pair

$$(c_0, g) \in \{]0, m_0[\times]0, \infty[\cup(m_0, 0)\}$$

The path in which $(c_0, g) = (m_0, 0)$ is the maximin (sustained) path. It has no growth. A path in which $(c_0, g) \in \{]0, m_0[\times]0, \infty[\}$ (so that $c_0 < m_0$ and $g > 0$)

maximin value. For example, the path followed could be a logistic growth curve. This path would be more difficult to solve than the path proposed in the text but would give no more insight into the problem. Considering quasi-arithmetic growth is another example. In this case, there is no limit to growth, or more precisely, the dynamic limit represented by the maximin value increases forever along with consumption.

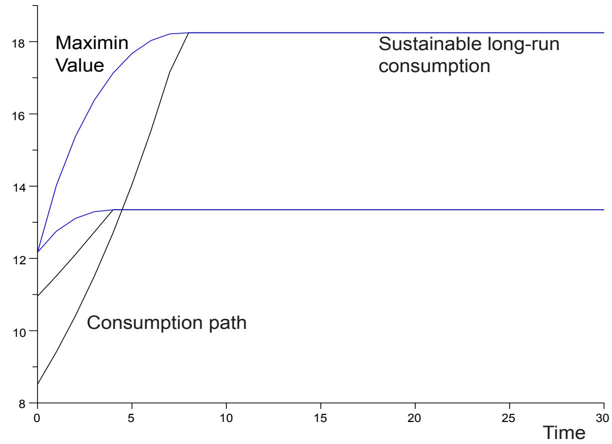

Figure 5: Exponential consumption and Maximin value function

has growth. However, growth at a constant rate cannot go on forever. There is an endogenous time $T(S_0, K_0, c_0, g)$ at which consumption catches up to the dynamic maximin value indicator, i.e., $c(T(\cdot)) = c_0 e^{gT(\cdot)} = m\left[S\left(T\left(\cdot\right)\right), K\left(T\left(\cdot\right)\right)\right]$. From then on, growth is no longer sustainable, and the level of consumption must remain at the maximin level; i.e., for $t \geq T(.)$, sustainability implies that $c\left(t\right) = m\left(S\left(T\right), K\left(T\right)\right)$.

At time T, only the part of the resource–allocation mechanism that drives the level of consumption changes, from allowing consumption to grow at rate g to keeping consumption constant at $c_\infty = m\left(S\left(T\right), K\left(T\right)\right)$. In this example of sustained growth with a constant growth rate pattern, the generations from time T stop sacrificing for sustainability improvement and enjoy the high, sustainable consumption level reached. Resource use is still determined by the maximin efficiency definition.

This analysis raises the question of optimality and efficiency. One may view society as making a choice according to a preference ordering $\mathcal{P}\left(c_0, g, c_\infty\right)$, by which initial consumption, the rate of growth and the very long-run, sustained consumption are evaluated. Fig. 6 depicts a convex-concave correspondence from the initial pair (S_0, K_0) to the attainable frontier, $\{(c_0, g, c_\infty)$ feasible from $(S_0, K_0)\}$. Growth is possible only if $c_0 < m(S_0, K_0)$.[12] For a given growth rate, a lower level of initial consumption allows a higher long-run level. For a given initial consumption, a lower growth rate allows a higher long-run consumption (as the actual consumption catches the maximin level more slowly). Given the initial level of consumption c_0, there is a trade-off between the eventual maximin consumption that is sustained after time T and the rate of growth that is sustained up to that level. A level of present consumption that is closer to the maximin value $m(S_0, K_0)$ entails a lower prospect for growth.

3.2.4 Intertemporal efficiency issues

The previously described resource-allocation mechanisms are not designed to be intertemporally efficient, and other paths could result in a higher utility for all generation.

We here consider a problem which does not consist in the maximization of current sustainability improvement by successive generations, but in the maximization of sustainability improvement over a time-span $[0, T]$, for a given T, along a given consumption path $\hat{c}(t)$. The intertemporal path of extraction has to be optimized then. This problem reads

$$\begin{aligned} \max_{r(\cdot)} \quad & \int_0^T \dot{m}(K, S, r, c) dt \\ s.t. \quad & \dot{K}(t) = K(t)^\alpha r(t)^\beta - \hat{c}(t) \\ & \dot{S}(t) = -r(t) \\ & (K(0), S(0)) = (K_0, S_0) \end{aligned} \tag{17}$$

This is equivalent to maximizing the sustainable utility at the end of the period $m(K(T), S(T))$.

It can be shown[13] that the optimal extraction rule of this problem must sat-

[12] "Negative growth" ($g < 0$) is required if $c_0 > m(S_0, K_0)$.

[13] Mathematical details are available upon request.

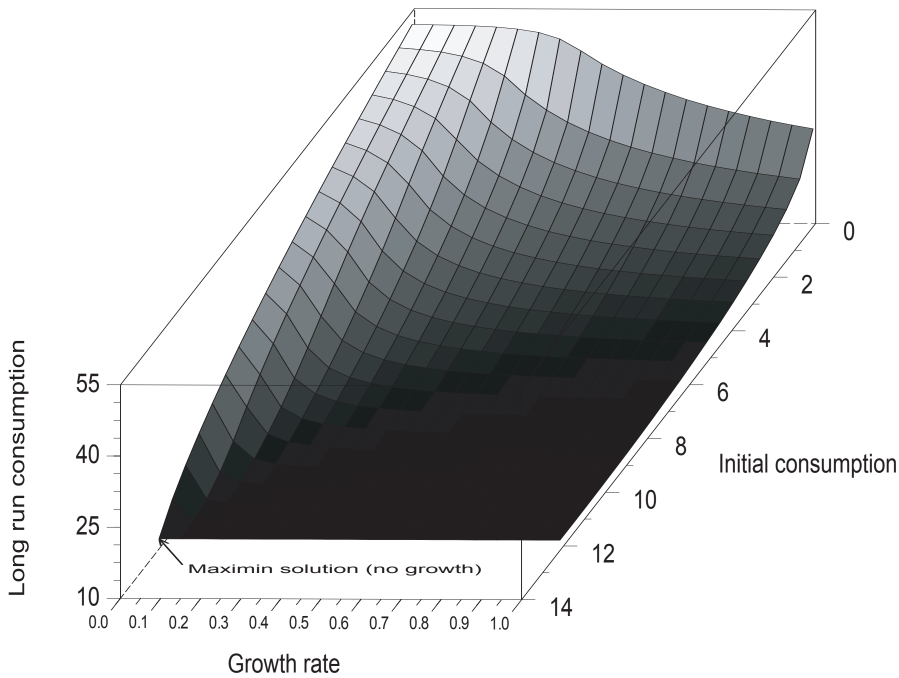

Figure 6: Necessary trade-offs between initial consumption, growth rate, and long-run (sustained) consumption in the DHS model

isfy Hotelling's intertemporal efficiency rule. Deriving the actual optimal extraction requires making an assumption on the growth pattern, and most cases are mathematical untractable.

4 Conclusion

We stress a property of a growth path that is not stressed by proponents of sustainable growth out of poverty. If the maximin path is not pursued, but instead some growth path is followed, then earlier generations must be deprived in order to divert toward investment the resources needed to sustain growth. Whether this deprivation is consistent with the vague criterion enunciated in the Brundtland report in terms of “needs” is not obvious. Growth is possible only at a cost. Open access, which in abstract terms is the main environmental problem facing humanity, is an inefficiency that cannot be overcome without current sacrifice. Growth is possible only within limits given by the technology and the environment. Otherwise, it can cause overshooting.

In this paper, we proposed a first analysis of the trade-offs between current consumption sacrifice (with respect to the maximin value) and sustainability improvement. A sacrifice makes room for growth. There is, however, an endogenous limit to growth along a given growth pattern.

Future developments of this working paper will include the analysis of efficient strategies to improve sustainability, as well as a discussion on social choice (in particular in a recursive context) and potential criteria to determine an optimal sustainable development pattern.

A Appendix

A.1 Mathematical details for the DHS model

Sustainability improvement maximizing extraction rule By differentiating the maximin value function (eq. 13) logarithmically with respect to time, we express the rate of growth of the maximin value as

$$\frac{\dot{m}}{m} = \left[\frac{\alpha-\beta}{1-\beta}\frac{\dot{K}}{K} + \frac{\beta}{1-\beta}\frac{\dot{S}}{S}\right] = \left[\frac{\alpha-\beta}{1-\beta}\frac{(K^{\alpha}r^{\beta}-c)}{K} - \frac{\beta}{1-\beta}\frac{r}{S}\right]. \tag{18}$$

Taking the derivative of this expression with respect to r and equalizing to zero gives us the extraction rule $\hat{r}(K,S)$ that maximizes the rate of growth of the maximin value (whatever is the consumption):

$$\hat{r}(K,S) = (\alpha-\beta)^{\frac{1}{1-\beta}} S^{\frac{1}{1-\beta}} K^{-\frac{1-\alpha}{1-\beta}}\ .$$

A.2 Intertemporal efficiency in the DHS model

By writing down the optimal control problem (17), using the Pontriagyn maximum principle, we derive from the first order conditions that the optimal extraction rule satisfies Hotelling's intertemporal efficiency rule.

The full solution of the problem requires specifying a consumption path, and most cases we examine do not allow us to derive an closed-form analytical solution.

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